Introduction To Differentiation

"Change is the only constant in this world."

Definition of Differentiation

Introduction to Differential Calculus (Differentiation)

* Definition :- Rate of change of a quantity with respect to another quantity.

B (500Km) ~~50Km/hr~~ 10 hrs

$v_{avg} = \frac{500}{10}$

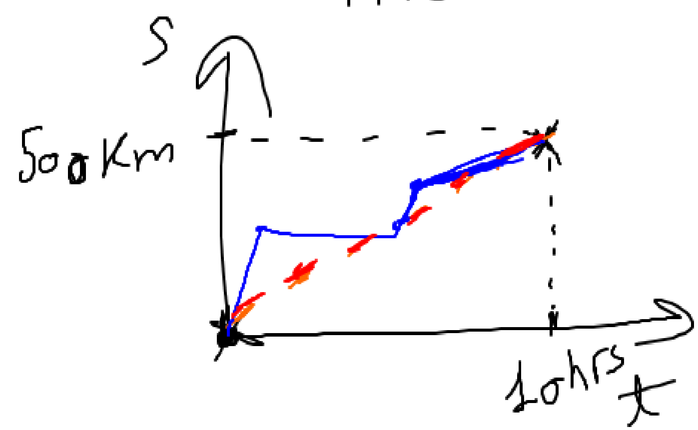
$v_{avg} = 50 Km/h$

$v_{avg} = \frac{\Delta x}{\Delta t} = \frac{S}{t} = \frac{\text{Displacement}}{\text{Time}}$

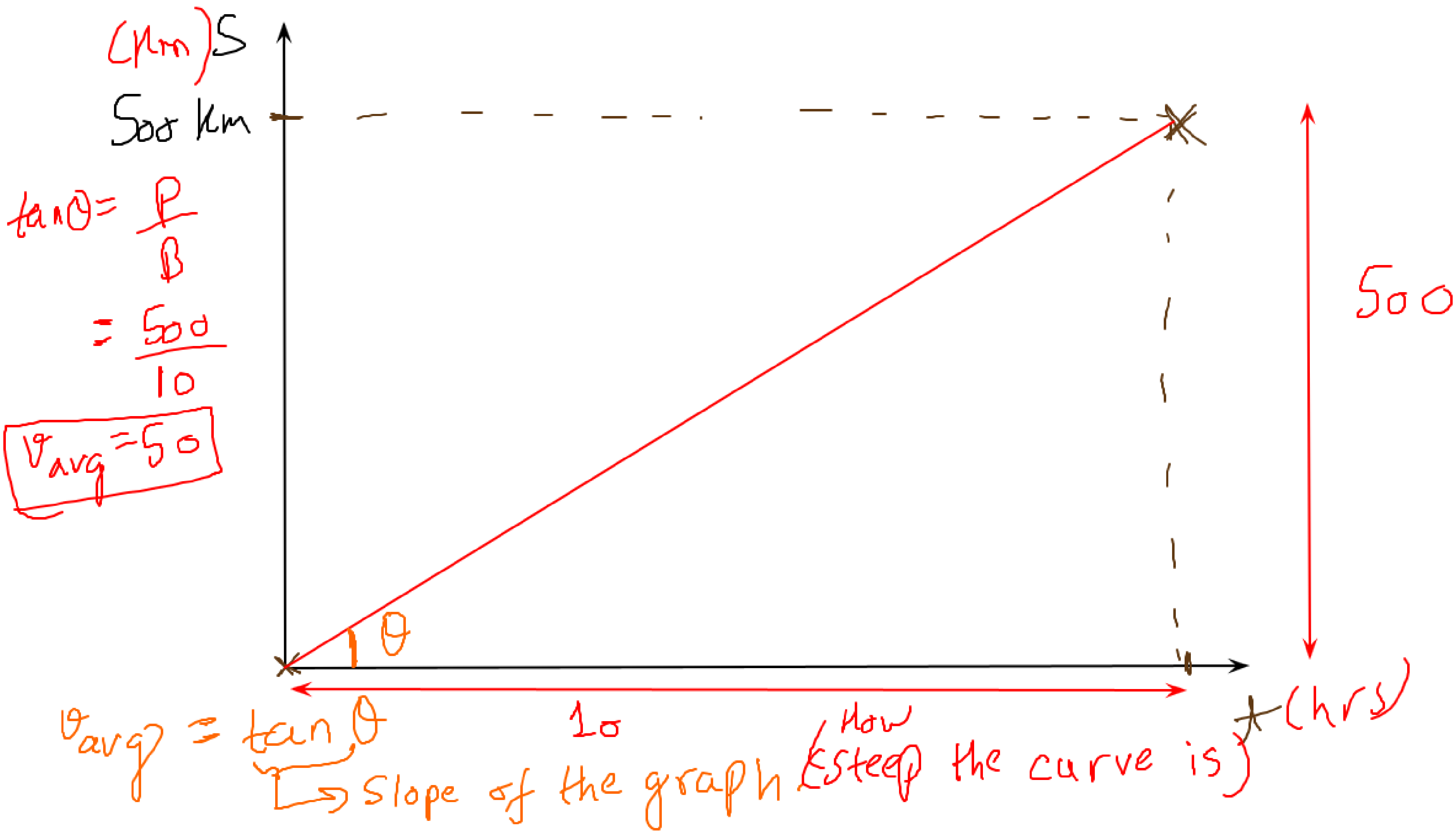
$v_{instantaneous}$ = At that instant

$v \to 0$

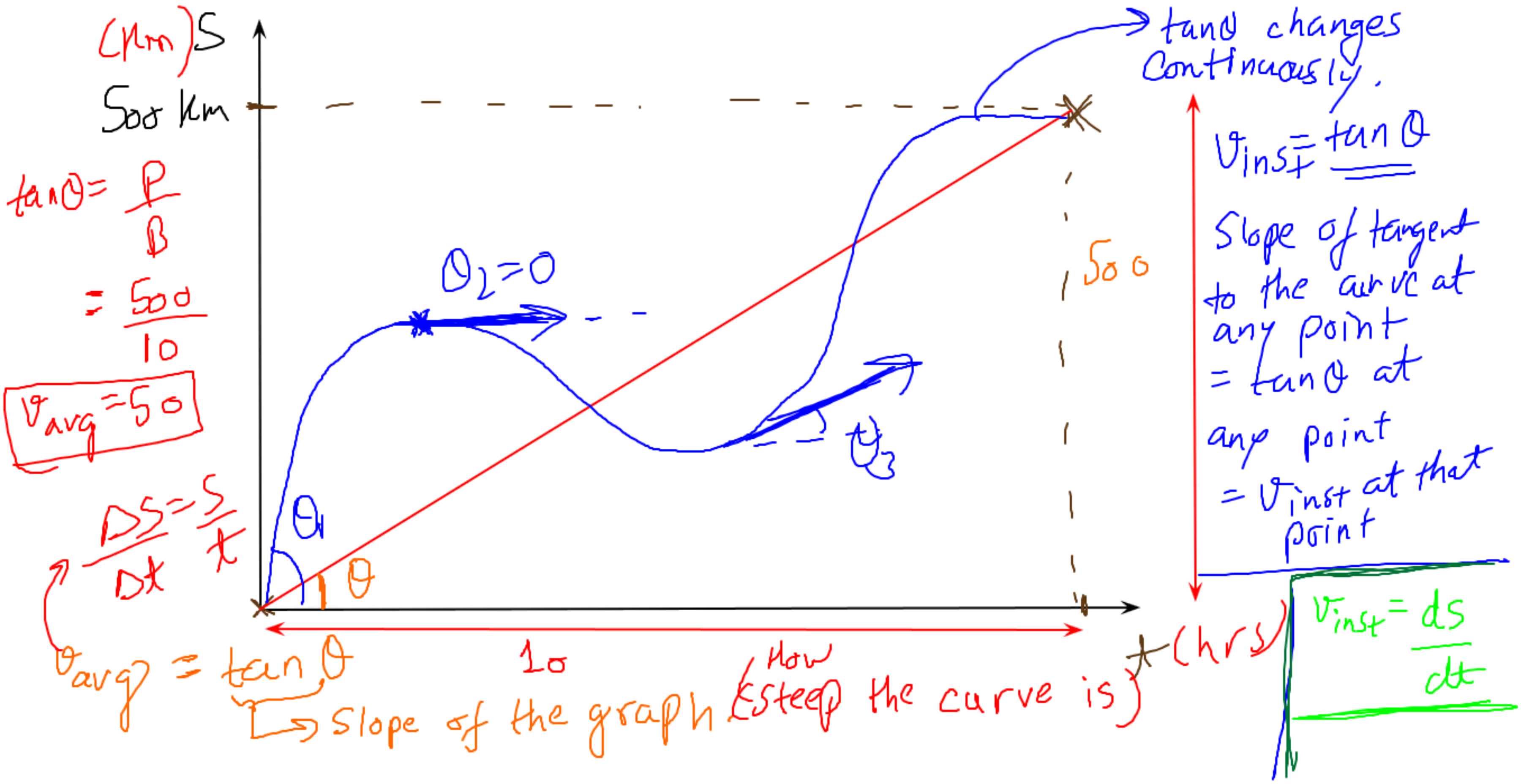
$v \to 120 Km/h$



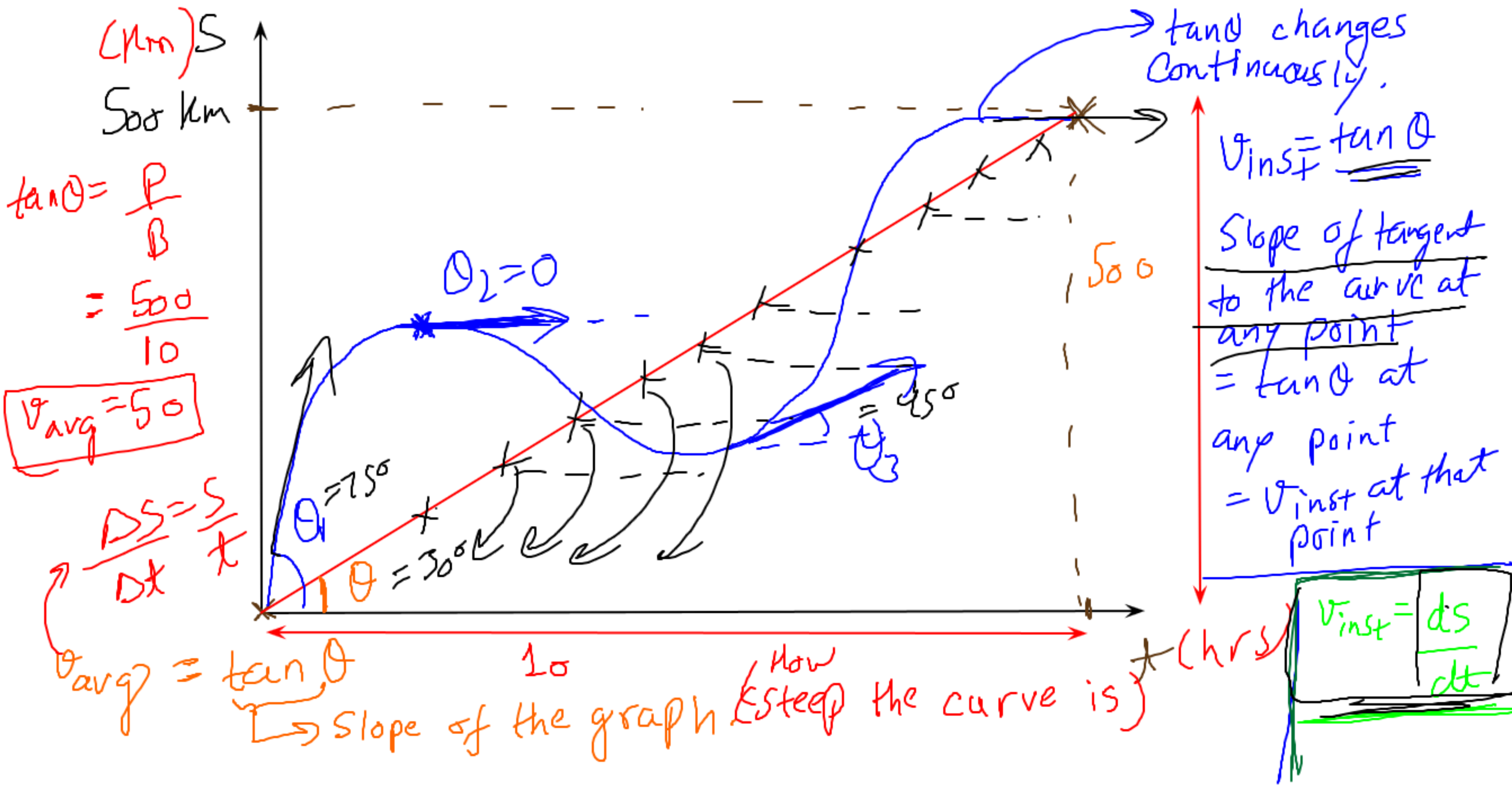
Slope of a Straight Line



Slope of a Curved Line

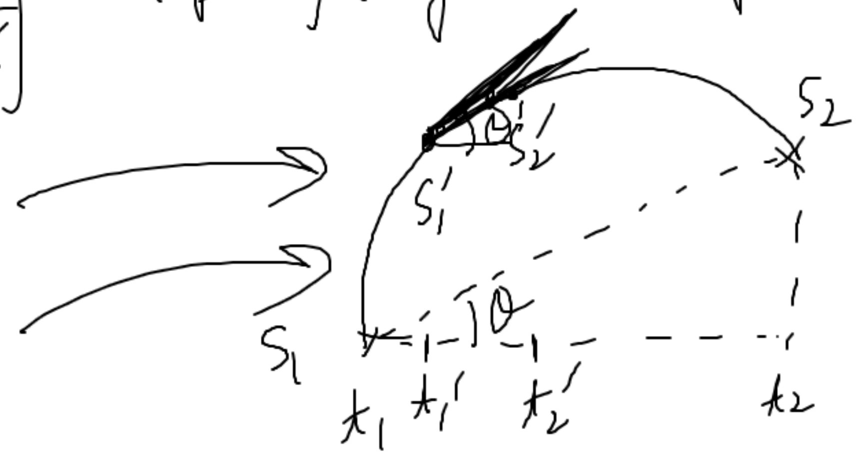


Slope of a Curved Line



Physical Interpretation of Slope

$\ast\ v_{inst} = \left(\frac{ds}{dt}\right)$ = Slope of tangent at a point $= \tan\theta$



$\tan\theta = \frac{s_2 - s_1}{t_2 - t_1}$

$\tan\theta' = \frac{s_2' - s_1'}{t_2' - t_1'}$

$\left(\frac{ds}{dt}\right) = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t}$

$\tan\theta = \left(\frac{ds}{dt}\right)$

→ Rate of change of displacement at any time w.r.t. time

Various Cases of Slope

$$v_{inst} = \left(\frac{ds}{dt}\right) \leftarrow$$

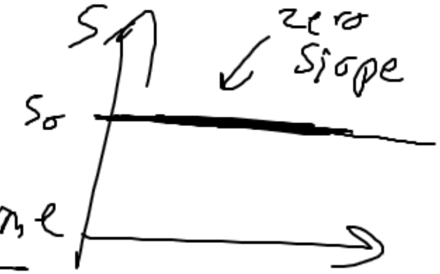
$$a_{inst} = \left(\frac{dv}{dt}\right)$$

$$a_{avg} = \frac{v_2 - v_1}{t} = \frac{\Delta v}{\Delta t}$$

$$a_{inst} = \frac{dv}{dt}$$

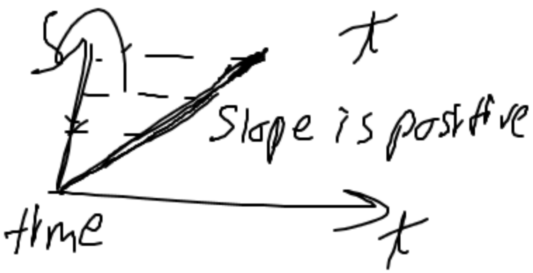
(i) If $\frac{ds}{dt} = 0$.

$S \rightarrow$ constant with time



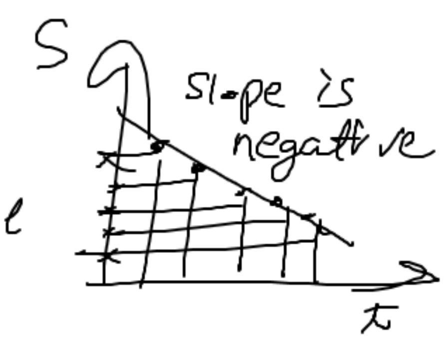
(ii) If $\frac{ds}{dt} > 0$

$S \rightarrow$ increasing with time



(iii) If $\frac{ds}{dt} < 0$

$S \rightarrow$ Decreasing with time



Interpretation of Slope (Graphical and Physical)

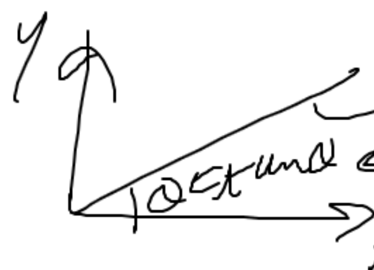
$\left(\frac{dy}{dx}\right)$ → Slope of y–x carve ($\tan\theta$) [Graphically]

↓

Rate of change of y w.r.t. x [Physically]

Sign of Slope

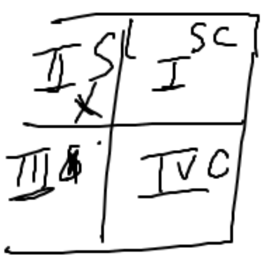
$* \frac{dy}{dx} \rightarrow$ Rate of change of y w.r.t x

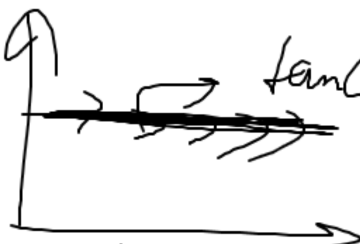


$\tan\theta = \frac{\sin\theta}{\cos\theta}$ ($\sin\theta \rightarrow$ +ve, $\cos\theta \rightarrow$ -ve)

If θ ↑, slope ↑, $\frac{dy}{dx}$ ↑, rate of change of y ↑

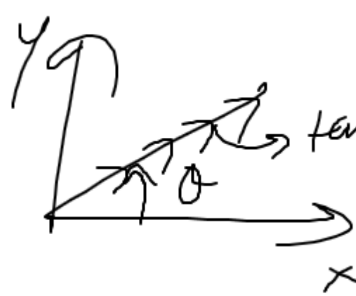
θ ↓ " ↓ " ↓ " ↓





$\tan\theta$ (slope) = Constant = zero

$\tan\theta < 0$



Basic Formulae in Differentiation

Basic Formulae in differentiation

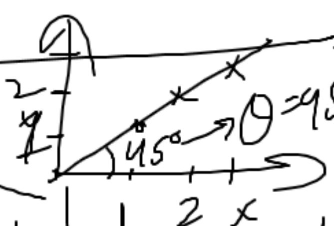
(i) Constant

$y = \text{constant} = K$

$$\frac{dy}{dx} = \frac{d(K)}{dx} = 0$$

(ii) $y = x$

$$\frac{dy}{dx} = \frac{dx}{dx} = 1$$



Slope $= m = 1$

(iii) $y = Kx$ ($K \rightarrow$ Any constant)

$$\frac{dy}{dx} = \frac{d(Kx)}{dx} = K\frac{d(x)}{dx} = K$$

eg.- $y = 3x$

$$\frac{dy}{dx} = ? = 3$$

(iv) $y = x^n$ ($n \rightarrow$ some constant)

$$\frac{dy}{dx} = \frac{d(x^n)}{dx} = n x^{n-1}$$

Basic Formulae in Differentiation

$y = x^n$ $\quad \frac{dy}{dx} = \frac{d(x^n)}{dx} = n\,x^{n-1}$

(i) $y = x^2 \rightarrow \frac{dy}{dx} = 2x^{2-1} = 2x$ ✓ ($n = 2$)

(ii) $y = x^3 \rightarrow \frac{dy}{dx} = 3x^{3-1} = 3x^2$ ✓

(iii) $y = x^4 \rightarrow \frac{dy}{dx} = 4x^3$ ✓

(iv) $y = x^5 \rightarrow \frac{dy}{dx} = 5x^4$ ✓

(V) $y = x \rightarrow \frac{dy}{dx} = \frac{dx}{dx} = 1\,x^{1-1}$

$= 1 . x^0 = 1$

(Vi) $y = 1 \Rightarrow \frac{dy}{dx} = \frac{d\,1}{dx} = \frac{dx^0}{dx}$

$= 0 . x^{0-1} = 0$

(Vii) $y = \sqrt{x} \Rightarrow \frac{dy}{dx} = \frac{d(x^{1/2})}{dx}$

$= \frac{1}{2} x^{(1/2 - 1)}$

$= \frac{1}{2} x^{\left(-\frac{1}{2}\right)}$

$\frac{1}{2\sqrt{x}} = \frac{1}{2x^{1/2}} \leftarrow$

Basic Formulae in Differentiation

(viii) $y = \frac{1}{x} \Rightarrow \frac{dy}{dx} = \frac{d(x^{-1})}{dx} = -1\, x^{-1-1} = \frac{-1}{x^2}$

(ix) $y = \frac{1}{\sqrt{x}} \Rightarrow \frac{d(x^{-1/2})}{dx} = \frac{-1}{2} x^{-\frac{1}{2}-1} = \frac{-1}{2} x^{-3/2} = \frac{-1}{2x^{3/2}} = \frac{-1}{2x\sqrt{x}}$

(x) $y = \left(\frac{1}{x^{5/3}}\right) \Rightarrow \frac{d(x^{-5/3})}{dx} = \frac{-5}{3} x^{-\frac{5}{3}-1} = \frac{-5}{3} x^{-8/3} = \frac{-5}{3x^{8/3}}$

$\frac{d}{dx} x^2 = 2x$

(xi)* $y = x^2 + 2x + 1 = (x+1)^2$

$\frac{dy}{dx} = ??\ \longrightarrow\ \frac{d(x+1)^2}{dx} = 2(x+1)$

$\frac{d}{dx}(x+1)^2 = 2x$

$\frac{d}{dx}(x+2)^2 = 2x$

$\frac{d}{dx}(x+n)^2 = 2x$

Thank You!